**Our Approach to The Problem:**

To find the product consisting of maximum number of items that fits the definition in the question, we decided to create a program that generates all possible hamburgers and checks them according to the rules in the question.

**Representation of Products in Our Program:**

A hamburger consists of 8 different items in this problem. However, number of pickles and tomatoes are defined to be equal in the problem. Also, the number of ketchup and lettuce are the same too. So, in order to represent a product, we used 6x1 matrices. Every row in those matrices represent the number of items.

|  |  |
| --- | --- |
| Variable | Item |
|  | Number of Beef Patty’s |
|  | Number of Buns |
|  | Number of Cheeses |
|  | Number of Onions |
|  | Number of Pickles  Number of Tomatoes |
|  | Number of Ketchups  Number of Lettuces |

**Generating Every Possible Product:**

In the definition of the problem, its clearly mentioned that a hamburger consists of the given items and it has to include at least one and at most five of those items. Since the number of free variables is six as mentioned in the previous topic, and each of those variables has to get one of five values, the number of all the possible hamburgers is 56, 15625. To Generate those hamburgers, we calculated the cartesian product of a set containing all the possible number of items as below. Then, we stored those values in the product matrix format described above.

15625 different matrices

**Calculating the Nutritional Values:**

To calculate the nutritional value of each matrix in order to check if that product complies with the demands or not, we created three 1x6 matrices that contain the nutritional value of each ingredient/ingredients. For example, the first cell of the sodium matrix represents the amount of sodium in one unit of the first ingredient, which is sodium. So it contains 50 in that cell.

The last two cells in each nutrient’s matrix contains the sum of those nutrients for both ingredients they represent. For Example, the last cell of sodium matrix is 163, which is the sum of the amount of sodium in a lettuce and a ketchup.

After creating those matrices, to calculate the nutritional values of a product, all we need to do is calculate the dot product of product and given nutrient. For example, to calculate the total amount of sodium contained in the first product, we calculate the dot product of first product and sodium as below:

**Checking if the Product Meets the Nutrient Limitations:**

To check if the product meets the limitations, we simply run the procedure above for each single product and check if it meets the limitations. If it does, we store it in a python list. We found out that there are 5194 possible products that meet the limitations. After that, all we needed to do is calculate the number of ingredients in those products and select the one that contains the maximum number of ingredients.

**Calculating the Number of Ingredients in a Product:**

To calculate the number of ingredients in a product, we created a procedure which is similar to the procedure we apply to calculate the nutritional values. We created a matrix that represent the number of ingredients, 1 for rows that consist of only one ingredient and 2 for rows that contain two ingredients like the last two rows.

Later, when we calculate the dot product of this matrix and the product, we get the number of ingredients in that product. For Example, the operation below calculates the number of ingredients in the first product.

**Calculating the Cost of a Product:**

To calculate the cost of a single product, we again created a similar procedure. We created a 1x6 cost matrix that contains the price of each row. Again, the dot product of two matrices will result in the price of a given product. For example, below operation calculates the total cost of the first product.

**Results:**

After the procedures we described, our program found out that the product that both contains the highest number of ingredients and meets the limitations contains 31 ingredients. The product is represented below:

|  |  |
| --- | --- |
| Value | Item |
|  | Number of Beef Patty’s |
|  | Number of Buns |
|  | Number of Cheeses |
|  | Number of Onions |
|  | Number of Pickles  Number of Tomatoes |
|  | Number of Ketchups  Number of Lettuces |

So, the best product contains 4 Beef Patty’s, a bun, a cheese, 5 onions, 5 pickles, 5 ketchups, 5 lettuces, 5 tomatoes. Since there is no other product found by our program that both meet the limitations and has the same or higher number of ingredients, this product is our answer to the question.

**Total Nutritional Values and Cost of the Product:**

**Nutritional Values:** *Total Sodium Amount:*

*Total Fat Amount:*

*Total Calory Amount:*

**Cost:**

*Total Cost:*